10.5: Polar Coordinates

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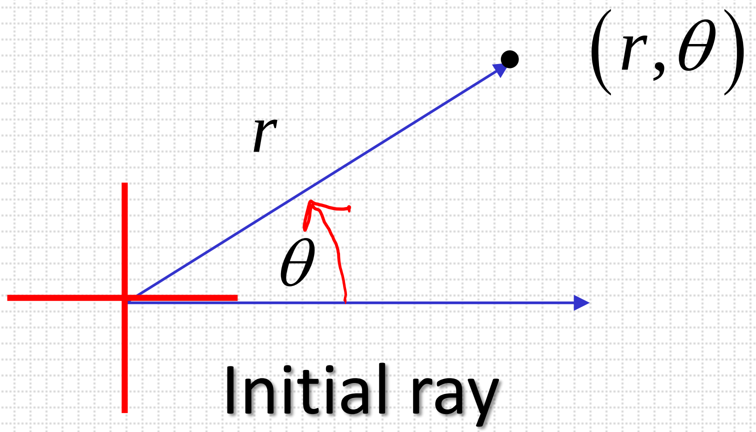
One way to give someone directions is to tell them to go three blocks East and five blocks South – like rectangular coordinates (x, y).

Another way to give directions is to point and say "Go a half mile in that direction."

Polar graphing is like the second method of giving directions. Each point is determined by a ***distance*** and an ***angle***.



A polar coordinate pair

$$\boxed{(r,\theta)}$$

determines the location of a point.

Goals:

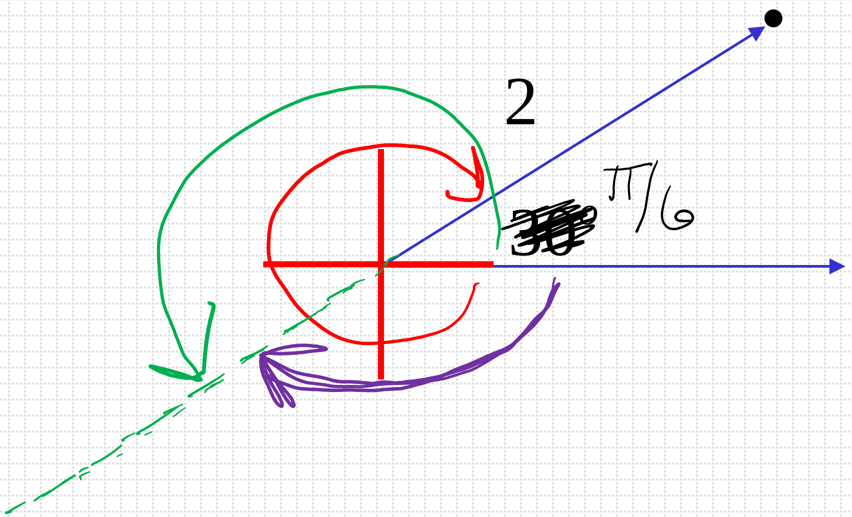
1. Given a point in rectangular coordinates, convert to polar (with restrictions.)

2. Given a point in polar coordinates, convert to rectangular.

3. Given an equation in rectangular coordinates, convert to polar.

4. Given an equation in polar coordinates, convert to rectangular.

More than one coordinate pair can refer to the same point – name the point with the given restrictions.



$r > 0, \theta > 0 \rightarrow \left(2, \frac{\pi}{6}\right)$

$r > 0, \theta < 0 \rightarrow \left(2, -\frac{11\pi}{6}\right)$

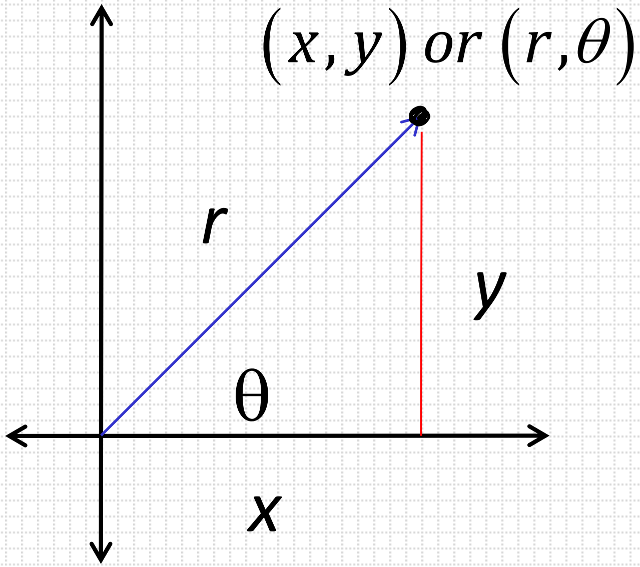
$r < 0, \theta > 0 \rightarrow \left(-2, \frac{7\pi}{6}\right)$

$r < 0, \theta < 0 \rightarrow \left(-2, \frac{-5\pi}{6}\right)$

All of the polar coordinates of this point are:

$$\left(2, \overset{\pi/6}{\cancel{30^\circ}} + n\cdot \overset{2\pi}{\cancel{360^\circ}}\right) \quad n \in \mathbb{Z}$$

Relationship between Polar and Cartesian Coordinates:



Generally speaking:

$$x = r\cos\theta$$

$$y = r\sin\theta$$

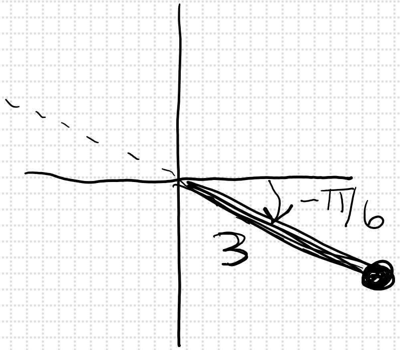
$$r^2 = x^2 + y^2$$

$$\theta = \tan^{-1}\left(\frac{y}{x}\right)$$

Plot the points with given polar coordinates and find their Cartesian coordinates.

$\tan\theta = 4/3 = y/x$

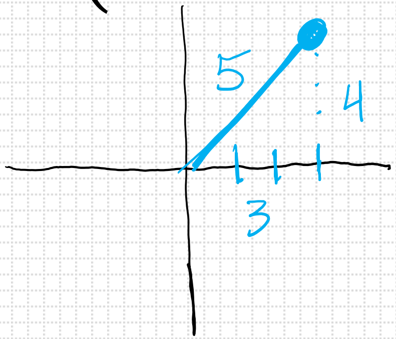
a. $\left(-3, \frac{5\pi}{6}\right) = \boxed{\left(\frac{3\sqrt{3}}{2}, \frac{-3}{2}\right)}$



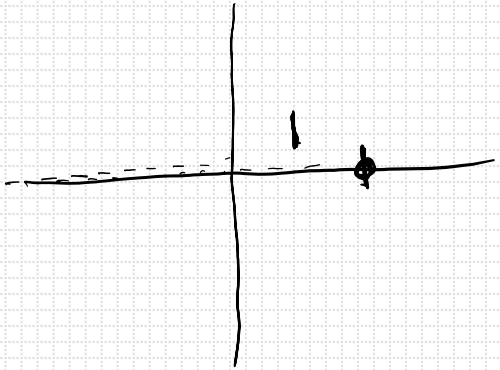
$x = r\cos\theta$
$= (-3)\cos\left(\frac{5\pi}{6}\right)$
$= (-3)\left(-\frac{\sqrt{3}}{2}\right)$

$y = r\sin\theta$
$= (-3)\sin\left(\frac{5\pi}{6}\right)$
$= (-3)(1/2)$

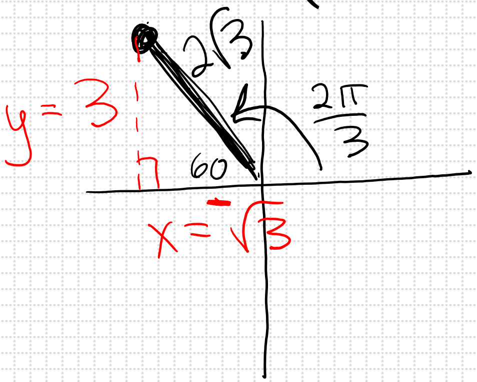
b. $\left(5, \arctan\left(\frac{4}{3}\right)\right) = (3,4)$



c. $(-1, 7\pi) = \boxed{(1,0)}$



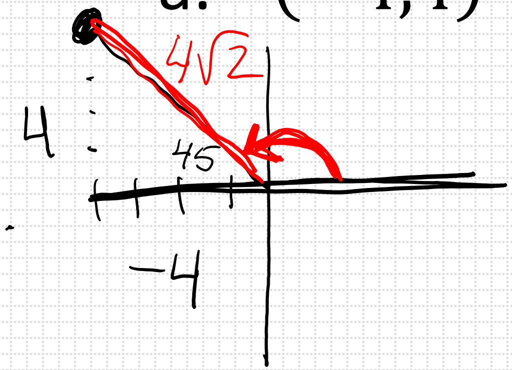
d. $\left(2\sqrt{3}, \frac{2\pi}{3}\right) = (-\sqrt{3}, 3)$



$x = r\cos\theta$
$= (2\sqrt{3})\cos\left(\frac{2\pi}{3}\right)$
$= (2\sqrt{3})(-1/2)$

Plot the points with the given Cartesian coordinates and convert to polar coordinates with $r > 0, \theta > 0$.

a. $(-4,4)$ $= \left(4\sqrt{2}, \frac{3\pi}{4}\right)$



$r^2 = (-4)^2 + (4)^2$

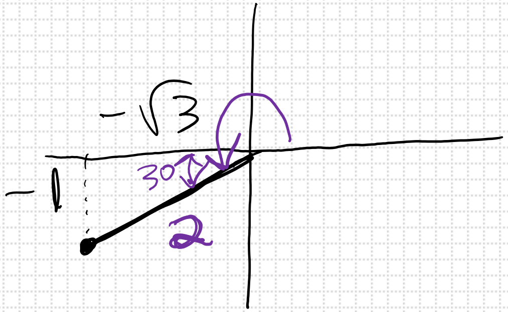
$r^2 = 32$

⋮

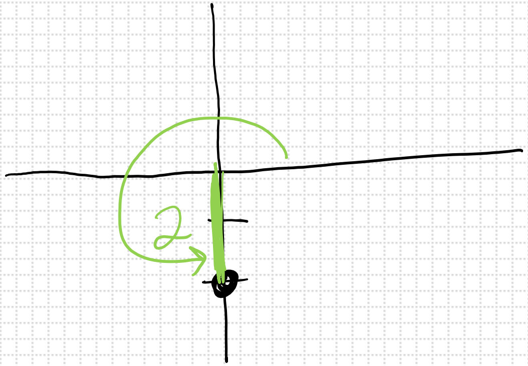
b. $(-\sqrt{3}, -1)$ $= \left(2, \frac{7\pi}{6}\right)$

$r^2 = (-\sqrt{3})^2 + (-1)^2$

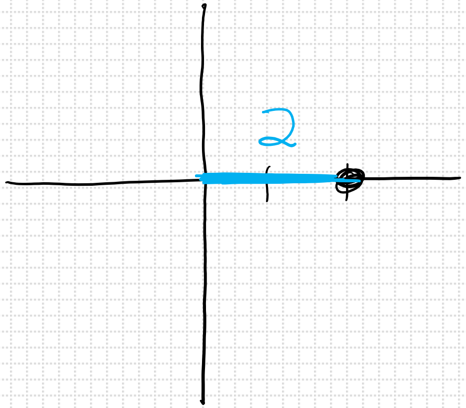
$r = 2$



c. $(0, -2)$ $= \left(2, \frac{3\pi}{2}\right)$



d. $(2,0)$ $= (2, 0)$



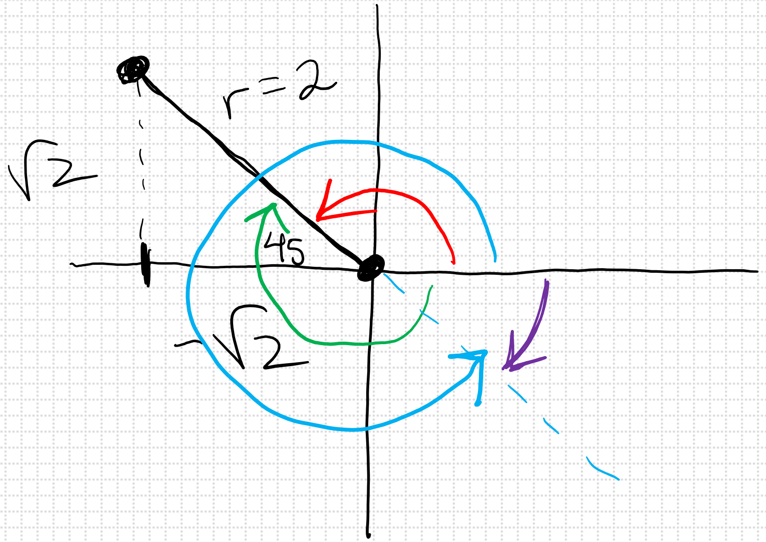
Rename the point $\left(-\sqrt{2}, \sqrt{2}\right)$ in polar with the given restrictions.

a. $r > 0, \theta > 0$ $\left(2, \frac{3\pi}{4}\right)$

b. $r > 0, \theta < 0$ $\left(2, \frac{-5\pi}{4}\right)$

c. $r < 0, \theta > 0$ $\left(-2, 7\pi/4\right)$

d. $r < 0, \theta < 0$ $\left(-2, -\pi/4\right)$



Rewrite the following cartesian *equations* in polar form.

$$\underbrace{x^2+y^2}=81$$

$$r^2=81$$

$$\boxed{r=9}$$

$$\underbrace{x^2+y^2}+10x=0$$

$$r^2+10r\cos\theta=0$$

$$\cancel{r}\left(r+10\cos\theta\right)=0$$

$$\boxed{r=-10\cos\theta}$$

$$y=7$$

$$r\sin\theta=7$$

$$\boxed{\begin{gathered} r=\frac{7}{\sin\theta} \\ r=7\csc\theta \end{gathered}}$$

$$x^2-y^2=1$$

$$(r\cos\theta)^2-(r\sin\theta)^2=1$$

$$r^2\cos^2\theta-r^2\sin^2\theta=1$$

$$r^2\left(\cos^2\theta-\sin^2\theta\right)=1$$

$$\boxed{r=\frac{1}{\pm\sqrt{\cos^2\theta-\sin^2\theta}}}$$

Rewrite the following polar *equations* in cartesian/rectangular.

$$r = 5\sec\theta = \frac{5}{\cos\theta}$$

$r\cos\theta = 5$

$\boxed{x = 5}$

$$r = \frac{6}{2-\cos\theta} \qquad \boxed{3x^2+4y^2-12x-36=0}$$

$2r - r\cos\theta = 6$

$2r - x = 6$

$(2r)^2 = (x+6)^2$

$4r^2 = x^2+12x+36$

$4x^2+4y^2 = x^2+12x+36$

$$r\left(r = 3\cos\theta\right)$$

$r^2 = 3r\cos\theta$

$x^2+y^2 = 3x \Rightarrow \boxed{x^2+y^2-3x=0}$

$$r = \sec\theta\tan\theta$$

$r = \frac{1}{\cos\theta}\cdot\frac{\sin\theta}{\cos\theta}$

$r\cos\theta = \tan\theta$

$x = \frac{y}{x}$

$\boxed{x^2 = y}$

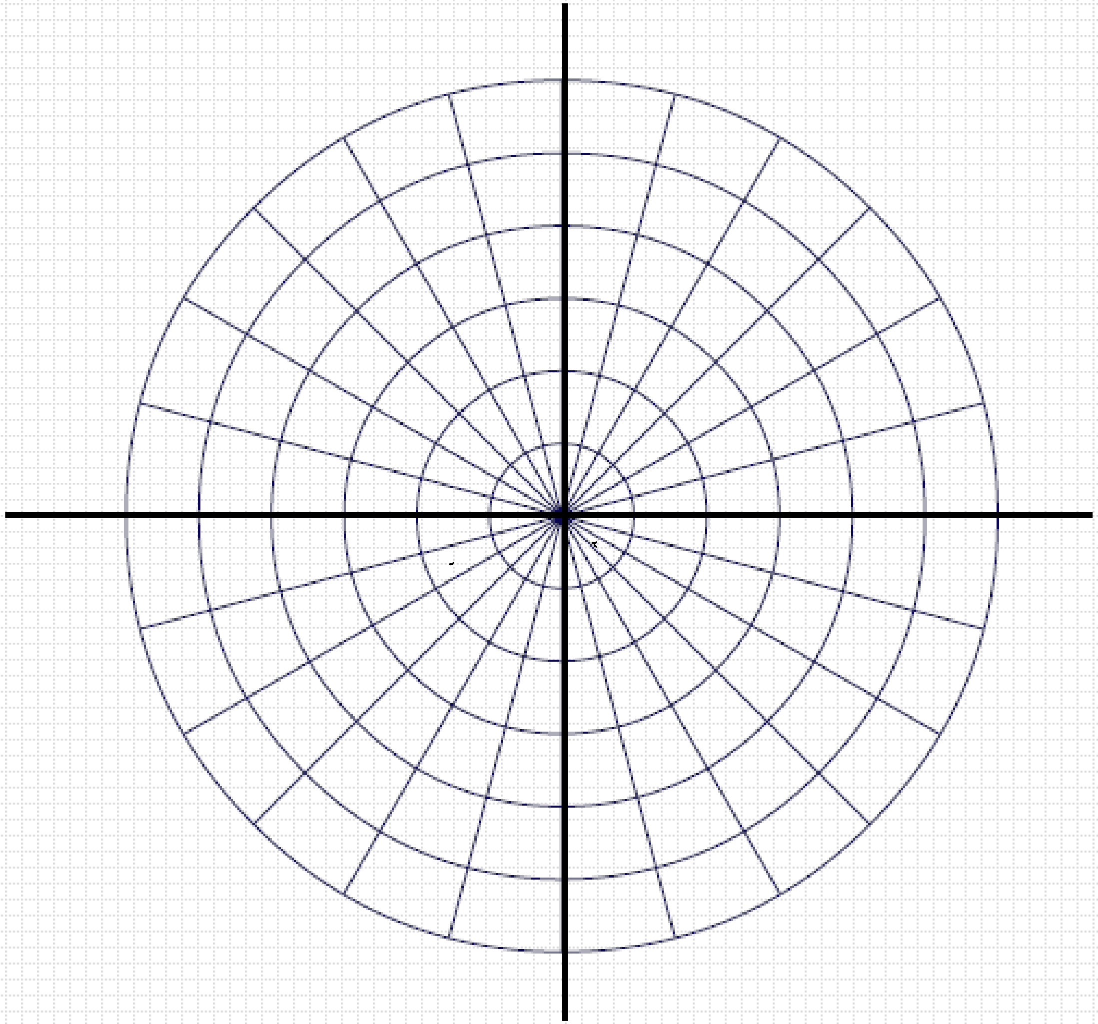
Homework:

Anton Handout

- #1 – 5 every other part (a, c, e)
- # 6 all
- # 9 – 12 (left column)

Graph the following. Then convert to Cartesian coordinates.

$$r = \sin\theta$$



Graph the following. Then convert to Cartesian coordinates.

$$r = \cos 2\theta$$